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# A new approach to automated study of isoptic curves

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Let $\mathcal{C}$ be a plane curve. For a given angle $\theta$ with $0 \leq \theta \leq 180^\circ$), a $\theta$-*isoptic* of $\mathcal{C}$ is the geometric locus of points in the plane through which pass a pair of tangents with an angle of $\theta$ between them. The special case for which $\theta = 90^\circ$ is called an *orthoptic curve*. The orthoptics of conics are well known: the directrix of a parabola, the director circle of an ellipse, and the director circle of a hyperbola (in this case, its existence depends on the eccentricity of the hyperbola).

Orthoptics and $\theta$-isoptics can be studied for other curves, in particular for closed smooth convex curves; see [1]. Isoptics of an astroid are studied in [2] (see Figure 1) and of Fermat curves in [3]. If $\mathcal{C}$ is an astroid, there exist points through which pass 3 tangents to $\mathcal{C}$, and two of them are perpendicular. These works combine geometrical experimentation with a Dynamical Geometry System (DGS) GeoGebra and algebraic computations with a Computer Algebra System (CAS). For them, the curve has been defined by a parametrization. A new

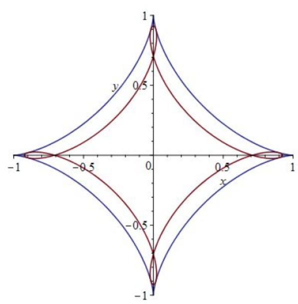


Figure 1: The 45-isoptic of the astroid

approach to these curves is proposed, using the DGS GeoGebra, not only its geometrical part but also its CAS component. The central feature is the connection between the two components of the same software package, enabling automatic switching between different registers of representation. This approach enables to determine the $\theta$-isoptics of various curves, either closed or not. Moreover, the dynamics of the work is essential for the study of the convexity of the $\theta$-isoptic. Students, teachers and researchers can make their own experiments, checking the existence of flexes, changing curves to look for invariant properties, etc.

We demonstrate this approach with GeoGebra applets [4] and [5] for parabolas and other planes curves, either closed or not. Here there is no need to use parametric equations for defining $\mathcal{C}$, and the work is based on implicit equations.

This automated work allows undergraduates to be acquainted with an advanced topic in Differential Geometry.

**Keywords:** Plane curves, isoptics, automated proof, dynamical geometry

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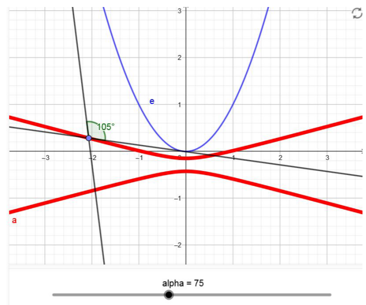


Figure 2: A screenshot of an applet

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